

MA 351: Elementary Linear Algebra

Spring 2019

1. Systems of linear equations, elementary row operations, Gaussian elimination;
2. Matrix algebra;
3. Finite dimensional vector spaces and subspaces, linear combination, linear dependence and independence, basis and dimensions;
4. Linear transformations, surjectivity and injectivity of transformations, inverse of transformation and matrices
5. Determinants, their computations and properties;
6. Eigenvalues and eigenvectors, diagonalization process.

Textbook: R. Penney: *Linear Algebra*.

MA 351: Elementary Linear Algebra
Spring 2019, Purdue University
<http://www.math.purdue.edu/~yip/351>

Course Description:

Systems of linear equations, matrices, finite dimensional vector spaces, determinants, eigenvalues and eigenvectors, inner products and orthogonality

Instructor:

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Contact Information:

Office: MATH 432
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Lecture Times and Places:

Section 112 (CRN 15967): T, Th 09:00 - 10:15, UNIV 217

Office Hours:

Tue: 2:00-3:00pm, Wed: 4:00-5:00pm, or by appointment.

Textbook:**Main Text (required):**

[P] *Linear Algebra, Ideas and Applications*, 4th edition, Richard Penney, Wiley.

You are highly encouraged to make good use of the textbook by reading it.

Homework:

Homeworks will be assigned weekly, due usually on Thursday in class. They will be gradually posted as the course progresses. Please refer to the course announcement below.

- **Steps must be shown to explain your answers. No credit will be given for just writing down the answers, even if it is correct.**
- **Please staple all loose sheets of your homework to prevent 5% penalty.**
- **Please resolve any error in the grading (hws and tests) WITHIN ONE WEEK after the return of each homework and exam.**

- **No late homeworks are accepted (in principle).**
- **You are encouraged to discuss the homework problems with your classmates but all your handed-in homeworks must be your own work.**

Examinations:

Tests: Midterm One and Midterm Two, in class, dates TBA

Final Exam: During Final Exam Week

No books, notes or electronic devices are allowed (nor needed) in any of the tests and exam.

Grading Policy:

Homeworks (25%)

Test (40%, 20% each test)

Final Exam (30%)

Class Participation (daily or weekly quizzes, etc, 5%)

You are encouraged to attend all the lectures. However, I do not take attendance. The quizzes are used to check your basic understanding and provide opportunity for you to mingle with your classmates and myself. It is open book, open note and open discussion, hopefully a fun activity. No make-up quiz will be given. You do not need to worry if you miss a few. However, if you anticipate to miss more (for legitimate reasons), please by all means let me know **as soon as possible.**

You are expected to observe academic honesty to the highest standard. Any form of cheating will automatically lead to an F grade, plus any other disciplinary action, deemed appropriate.

Accommodations for Students with Disabilities and Academic Adjustment:

University policy and procedures will be followed.

For more detail information, please click [ADA Information](#)

Course Outline (tentative):

Chapter 1: linear systems and their solutions, matrices;

Chapter 2: vector spaces and subspaces, linear (in)dependence, dimension;

Chapter 3: linear transformation;

Chapter 4: determinants;

Chapter 5: eigenvectors and eigenvalues;

Chapter 6: orthogonality

Course Progress and Announcement:

(You should consult **this section** regularly, for homework assignments, additional materials and announcements.)

Key outcomes of this course.

- (1) setting up of systems of linear algebraic equations, finding their solutions, interpretation of solutions;
- (2) effective use of matrix notations and their interpretation;
- (3) interpretation of (1) and (2) using the concepts of abstract (and yet concrete and useful) vector spaces, in particular, basis, dimension, and geometry of subspaces;
- (4) last but not least, understanding and appreciation of the need of giving proofs, how to write proofs and knowing what constitutes a proof.

Reading of the textbook. I highly encourage you to read the textbook.

I will try to follow the materials from the textbook, but inevitably there will be deviations: in the presentation styles, emphasis, examples, and solution methods. Going to the lectures and reading the textbook can thus give you multiple viewpoints on the materials. In addition, the textbook has many worked out examples.

Use of technology. You are highly encouraged to try and experiment with technology, in particular Matlab. (I often use it myself.)

My **motto** on the use of technology:

If technology helps you understand, by all means use it. Otherwise, use it at your own risk.

Beware that during the tests and exam, no technology will be allowed.

Some matlab information.

- (1) **Matlab and linear algebra go hand in hand.** Its effective usage
 - (a) requires good understanding of linear algebra, and also
 - (b) enhances your understanding of linear algebra.
- (2) [A very simple tutorial](#). Just follow the steps in the file.
- (3) There are "lots" of Matlab manual available online. Type "matlab manual" in google.

NOTATION MATTERS!!!!!!

The notations created for and used in linear algebra are supposed to make the concepts and computation easier.

But you need to **UNDERSTAND** them in order to get the most out of them.

Week 1:**Tues, Jan 8:**

[P 1.2] Geometric interpretations of finding solutions:

- (i) (row) intersection between lines, planes;
- (ii) (column) writing vector as linear combination.

[NOTE: Two interpretations of solving linear systems](#)

Thur, Jan 10:

[P 1.2] Elementary row operations; three possibilities of scenarios:

- (i) **unique solution;** (ii) **infinitely many solutions;** (iii) **no solutions.**

Homework 1: due Thursday, Jan 17, in class.

(Unless otherwise stated, all homework problems are from the textbook, Penney, Linear

Algebra, 4th Edition.)

p.39: #55, 59;

p.64: #67, 69.

Week 2:**Tues, Jan 15:**

[P 1.3] Elementary row operations, Gaussian elimination.

[NOTE: Examples on solving \$m \times n\$ systems](#)**(m - number of equations - might not equal n - number of unknowns)****Thur, Jan 17:**

[P 1.3] Key concepts coming out of Gaussian eliminations:

elementary row operations (ERO),

equivalence between systems (under ERO),

row echelon form (REF),

backward substitution,

pivot vs free variables,

reduced row echelon form (RREF).

Three possibilities upon solving $m \times n$ linear systems:**(i) unique solution (only pivot variables, i.e. no free variables);****(ii) infinitely many solutions (some free variables);****(iii) no solution (inconsistent)**[NOTE: Gaussian Elimination](#)

Some applications: interpolating polynomials, traffic flows.

Homework 2: due Thursday, Jan 24, in class.[Homework 2](#)**Week 3:****Tues, Jan 22:**[P 1.1] Vectors in \mathbb{R}^n :**vector addition, scalar multiplication**, and their properties;**linear combinations** and **span**.[NOTE: \(General\) Vector Space](#)**Thur, Jan 24:**

[P 1.1] polynomials, functions, matrices as vector spaces;

[P 1.4] **matrix multiplied by a column vector, NOTATION MATTERS:** $A(X+Y) = AX + AY$; $A(aX) = aAX$; $(A+B)X = AX + BX$;linear system of equation in matrix form: **$AX=b$** ;**Column space of A, $\text{Col}(A)$: $AX=b$ is solvable if and only if b belongs to $\text{Col}(A)$.****Homework 3: due Thursday, Jan 31, in class.**

p.17: #3, 5, 26;

p.86: #96, 97, 106, 107, 108, 109, 111, 112, 121.

Week 4:

Tues, Jan 29:

[P 1.4] **homogeneous** ($AX=0$) vs **inhomogeneous** ($AX=b$) systems;

Null space of a matrix A ($\text{Null}(A)$): solution of the homogeneous system.

Structure of solutions for $AX=b$ (assume it is consistent): $X = p + \text{Null}(A)$,

where p is a **translation vector**, or a **particular solution**.

[NOTE: Column and Null spaces of a matrix](#)

[P 1.1] **linear dependence** and **redundant vectors**:

how to determine if a vector is redundant.

Thur, Jan 31:

[P 2.1] **linear dependence** and **independence**;

How to eliminate **all** redundant vectors.

[NOTE: Linear Dependence and Linear Independence](#)

Homework 4: due Thursday, Feb 7, in class.

p.108: #2.1, 2.3, 2.7, 2.11, 2.13, 2.17, 2.18.

Week 6:

Test One: Feb 14, in class.

Materials covered: Chapter 1 to Chapter 2.2.

The best way to review is to (i) go over lecture materials, (ii) read the textbook, and (iii) go over the homework and quiz problems.

No calculator or any electronic devices are allowed (nor needed.)

[Past Exam One](#)

[Solution of Quiz 1-5](#)

[Solution of Hw 1, 2, 3, and 4](#)

Test One Statistics: [\(Solution\)](#)

Total number of students = 45

A ($80 \leq \text{scores} \leq 100$): No. of students = 15 (33%)

B ($60 \leq \text{scores} \leq 79$): No. of students = 16 (36%)

C ($40 \leq \text{scores} \leq 59$): No. of students = 9 (20%)

D ($20 \leq \text{scores} \leq 39$): No. of students = 4 (9%)

F (scores ≤ 19): No. of students = 1 (2%)

Note: the above cut-offs are very rough and simple cut-offs, purely based on the test scores. I have not considered the hws, and quizzes.

Homework 7: due Thursday, Feb 21, in class.

[Homework 5](#)

Week 7:**Tue, Feb 19:**[P 2.1, 2.2] [NOTE: Basis and Dimension](#)**More unknown theorem:**

In any linear system with m equations and n unknowns with $n > m$, there must be **at least one free variables**.

More equation theorem:

In any linear system with m equations and n unknowns with $m > n$, there must be **a vector B such that $AX=B$ is not solvable**.

Well-definedness (uniqueness of) dimension;

Dimension is the **maximum** number of **lin ind** vectors

Dimension is the **minimum** number of vectors that can **span**

In an n -dim space, any **n lin ind vectors must span**,

In an n -dim space, any **n vectors that span must be lin ind**,

Dimension is the **effective number of degree of freedom**

Thur, Feb 21:

[P 2.3] **Col, Null, and Row** spaces associated with a matrix.

$\dim(\text{Col}) = \text{number of pivots} = \mathbf{rank}$;

$\dim(\text{Null}) = \text{number of free var} = \mathbf{nullity}$;

Rank+Nullity = Total number of variables (Rank-Nullity Theorem)

relationship between rank and nullity with lineary independence;

relationship between rank and nullity with solvability and uniqueness of solution

Non-singular matrices, equivalent properties of non-singular matrices.

[P 1.4] **Subspaces**

Subspace is **closed under vector addition and scalar multiplication**

[NOTE: Col, Null and Row spaces](#)

Homework 6: due Thursday, Feb 28, in class.

p.143: #2.64, 2.65, 2.66(a,c), 2.67(a,c), 2.76, 2.78

p.92: #1.115, 1.116, 1.117, 1.118, 1.119, 1.120

Week 8:**Tue, Feb 26:**

[P 3.1] **Linearity properties:**

closed under vector addition and scalar multiplication:

definition of subspaces, matrix multiplication ($X \mapsto AX$),

Linear transformations ($X \mapsto T(X)$), examples: reflection, projection, rotations

Thur, Feb 28:

[P 3.1] Linear transformations given by matrix multiplications:

$T(X) = AX$: X (in \mathbb{R}^n) $\mapsto AX$ (in \mathbb{R}^m), where A is an $m \times n$ matrix.

matrix representation of T , how to find the matrix corresponding to T .

Homework 7: due Thursday, Mar 7, in class.

p.158: #3.1, 3.2, 3.5, 3.6, 3.10, 3.11, 3.12, 3.13, 3.15

p.173: #3.26, 3.27

Week 9:**Tue Mar 5, Thur Mar 7:**

[P 3.2] Matrix multiplications

$$C^{(m \times n)} = A^{(m \times l)} * B^{(l \times n)}$$

beware of dimension compatibility

connection to **composition of linear transformation**: $[TS] = [T][S]$ **In general, AB is not equal to BA** **Homework 8: due Thursday, Mar 21, in class.**

p.174: #3.34, 3.35, 3.36, 3.37, 3.41, 3.42, 3.44, 3.48, 3.49, 3.50, 3.52

(hint for 3.48, 3.49, 3.50: look at 3.52)

Additional problem #1: find the matrix P that projects vectors in \mathbb{R}^2 onto the straight line $y=mx$.Show that $P^2 = P$.Additional problem #2: find the matrix R that reflects vectors in \mathbb{R}^2 with respect to the straight line $y=mx$. Show that $R^2 = I$.**(Week 10: Spring Break)****Week 11:****Tue Mar 19:**

General theory of maps and functions,

onto (surjective) and **one-to-one (injective)** maps,**inverse** of a map (for onto and one-to-one maps) $A^{(m \times n)}$ is onto $\iff AX=Y$ is solvable for any $Y \iff A$ has m pivot (the maximum number) variables $\iff \text{Rank}(A)=m$; $A^{(m \times n)}$ is one-to-one $\iff AX=Y$ has unique solution (if solvable) $\iff A$ has no free variables $\iff \text{Rank}(A) = n$ **Thur Mar 21**

[P 3.3] Inverse of a matrix

 A has an inverse $\iff A$ is onto and one-to-one $\iff AX=Y$ is uniquely solvable for any $Y \iff$ $\text{Rank}(A)=m=n$ (hence A is necessarily a square matrix)finding A inverse by row operations

properties of inverse

[NOTE: Inverse of a matrix](#)[NOTE: Two applications of matrix: Leontief input-output model and graph theory](#)[Leontief, Input Out Economics, Scientific American, 1951](#)**Homework 9: due Thursday, Mar 28, in class.**

p. 190: #3.64(a to j), 3.72, 3.74, 3.75, 3.76, 3.77, 3.78, 3.87

p. 202: (Self-Study Questions) 3.4, 3.5(a,b)

For the graph (Figure 3.10) in p. 180, the connectivity matrix M and M^2 are as given in the text. Find also M^3 .

Using M^2 and M^3 , indicate:

(1) how many 2-step paths are there from D to B and D to D?

Draw these paths explicitly, each on a separate graph.

(2) how many 3-step paths are there from B to B and B to D?

Draw these paths explicitly, each on a separate graph.

Week 12:

Tue Mar 26:

[P 4.1, 4.2] Determinant of a square matrix

(I) computation using co-factor expansion;

(II) computation using row reduction.

Thur Mar 28:

[P 4.2, 4.3]

Properties of determinants

Applications of determinants:

computation of area of parallelogram and volume of parallelepiped

solution of $AX=B$ (for invertible, square matrix A) (Cramer's Rule)

Week 13:

Test Two: Apr 4th, in class

Materials covered: Chapter 2 to Chapter 3. (Note: The whole Chapter 2 is included.)

(Even though I will not specifically ask questions about the materials before Test One, I do not know of any concepts before Test One that will not be used for Test Two.)

The best way to review is to (i) go over lecture materials, (ii) read the textbook, and (iii) go over the homework and quiz problems.

No calculator or any electronic devices are allowed (nor needed.)

[Past Exam Two](#)

[Solution of Quiz 6](#)

[Solution of Quiz 7](#)

[Solution of Hw 5, 6, 7, 8, and 9](#)

Test Two Statistics: [\(Solution\)](#)

(Total number of students = 45)

A (80 \leq scores \leq 100): No. of students = 19 (42%) (Test One: 15, 33%)

B (60 \leq scores \leq 79): No. of students = 15 (33%) (Test One: 16, 36%)

C (40 \leq scores \leq 59): No. of students = 8 (18%) (Test One: 9, 20%)

D (20 \leq scores \leq 39): No. of students = 3 (7%) (Test One: 4, 9%)

F (scores \leq 19): No. of students = 0 (0%) (Test One: 1, 2%)

(Note: the above cut-offs are very rough and simple cut-offs, purely based on the test scores. I have not considered the hws, and quizzes.)

Homework 10. Due: Apr. 11, Thursday, in class

p.249, #4.1

p.259, #4.12, 4.15, 4.16, 4.17, 4.24, 4.25, 4.26

p.268, #4.34, 4.36, 4.41

Week 14:**Tue Apr 9:**

[P 4.2, 4.3] Properties of determinants and their applications.

Cramer's rule for solving $AX=b$ (for invertible square matrices)Formula for A^{-1} (if $\det(A)$ not equal to zero).

[P 5.1] Eigenvalue and eigenvectors:

 $AX=\lambda X$ (**X must not be the zero vector**)**Thur Apr 11:**[P 5.1] Eigenvalues and eigenvectors: $AX=\lambda X$ Examples with **distinct and repeated eigenvalues**Applications: computing $(A^n)Y$ **Homework 11. Due: Apr. 18, Thursday, in class**

p. 280: #5.3, 5.5, 5.10, 5.11, 5.12, 5.13, 5.14, 5.15, 5.16

Week 15:**Tue Apr 16:**

[P 5.1] distinct and repeated eigenvalues

Algebraic multiplicities (m_i) vs geometric multiplicities (g_i),

deficient/defective eigenvalues and matrices,

independence of eigenvectors with distinct eigenvalues,

an application: matrix powers and Fibonacci sequence.

Thur Apr 18:

[P 5.2] Diagonalizable matrices and diagonalization process

[P 5.3] Complex eigenvalues and eigenvectors

Week 16:**Final Exam: Thur, 05/02, 8am-10am, SC 239**